Fostering mathematical creativity in school: Selected approaches illustrated with examples

Ljerka Jukić Matić [1]

Abstract

Creativity is considered an important skill of the twenty-first century, alongside critical thinking, collaboration, and communication. By developing these skills, students can prepare for future jobs, unpredictable social challenges, and future technology use. Moreover, creative individuals have an advantage in the classroom because they can apply what they have learned in new contexts and create new connections between different knowledge and skills. Creativity has also been recognized in mathematics education as an important skill for the 21st century. Thus, the past decade has seen a significant increase in mathematical creativity research, far outpacing previous years' efforts. Open-endedness in classrooms is one of the most popular teaching approaches to foster mathematical creativity and divergent thinking. This paper, provides examples of classroom-applicable approaches that encourage mathematical creativity: multiple solution tasks, multiple outcome tasks, problem-posing, and triptych.

Keywords

Mathematical creativity, Multiple solution tasks, Multiple outcome tasks, Problem posing, Triptych.

---

[1] ljukic@mathos.hr
School of Applied Mathematics and Informatics, University of Osijek, Croatia
https://orcid.org/0000-0002-8947-6333

Jukić Matić, Lj. (2024). Fostering mathematical creativity in school: Selected approaches illustrated with examples. In E. L. Juárez Ruiz, & L. A. Hernández Rebollar (Eds.), *Tendencias en la Educación Matemática 2024* (pp. 37–54). Editorial SOMIDEM.
https://doi.org/10.24844/SOMIDEM/S3/2024/02-02

Introduction

Enhancing the development of 21st-century skills such as critical thinking, creativity, collaboration, or communication (Organization for Economic Co-operation and Development [OECD], 2019) is the best way to prepare students for jobs that do not yet exist, social challenges that cannot be foreseen, and the usage of technologies that have not yet been invented. Educators and researchers in mathematics education have recognized the importance of cultivating creativity as an essential 21st-century skill (Leikin & Sriraman, 2016). Creative individuals have an advantage in the classroom because they can apply what they have learned in new contexts and create new connections between different knowledge and skills (Leikin & Elgrably, 2022). Mathematics education has also recognized creativity as a vital skill for the 21st century (Leikin, 2009; Sriraman, 2005). Thus, the past decade has seen a significant increase in mathematical creativity research, much outpacing previous years' efforts. This increased focus has prompted educators and researchers to investigate identifying, developing, and refining tasks specifically tailored to foster and enhance mathematical creativity in the classroom. Researchers also investigated the advantages of creative-directed activities for students' cognitive abilities and affective characteristics (e.g., Schukajlow & Krug, 2014; Tabach & Levenson, 2018). Through these research efforts, the educational community attempted to uncover the complexity of mathematical creativity, to use this understanding to improve students' learning experiences and equip them with the skills needed to tackle the challenges of the 21st century. This paper aims is to provide examples of some classroom-applicable approaches that encourage mathematical creativity. Therefore, the questions guiding the literature search were: 1. What is mathematical creativity? 2. What approaches encourage mathematical creativity in school mathematics?

Mathematical creativity

The study of creativity dates back to the seminal work of American psychologist Guilford (1950), who investigated the essence of intelligence. He investigated divergent and convergent thinking and argued that both are necessary for creative problem-solving: convergent thinking refers to exploring of variability, whereas divergent thinking refers to creating variability. He characterized creativity as divergent thinking and the ability to generate multiple ideas, create new patterns, transform knowledge and meaning, or apply object functions. Divergent thinking, characterized by fluency, flexibility, originality, and elaboration, laid the foundation for subsequent assessments of creativity, most notably the Torrance Tests of Creative Thinking (Torrance, 1974, in Elgrably & Leikin, 2021). These tests sought to provide a quantitative measure of creativity, influencing various domains, including mathematics education. Components identified by Guilford and Torrance were also used to evaluate mathematical creativity; Silver (1997) refined the definition of creativity-enhancing problem solving and proposed the following components: fluency is developed by generating multiple ideas, multiple answers to a problem (when such exist), exploring situations, and raising multiple ideas; flexibility is advanced by generating new solutions when at least one has already been produced; novelty is advanced by exploring many solutions to a problem and generating a new one.

Some scholars, like Sriraman (2009), have drawn upon the mathematical creativity model proposed by mathematician J. Hadamard (1945, in Sriraman, 2009). Hadamard's model delineates mathematical creativity through four stages: preparation, incubation, illumination, and verification, offering a structured perspective on the creative process. The first stage involves putting great effort into getting insights into the problem. The second stage, incubation, refers to putting the problem aside and engaging the mind in solving other problems. The third stage, illumination, happens when the solution emerges suddenly while the individual does other unrelated activities. Finally, the last stage is comprised of verifying the result by making it precise and expressing these ideas in writing or oral form. During the last stage, verification, individuals seek other alternatives, extensions, or solutions. Hadamard constructed his model based on Wallas' gestal framework (1926, in Sriraman, 2009). This model is frequently utilized as a benchmark when interpreting mathematical advancements.

Nevertheless, a universally accepted definition of mathematical creativity still needs to be discovered due to the fact that definitions of mathematical creativity are constructed based on characteristics of general creativity (Bicer, 2021). Additionally, this impacted research in mathematics education, as various researchers directed their attention towards distinct facets of creativity. While there is a need for more agreement among scholars regarding the precise definition of mathematical creativity, certain common aspects have been identified. Bicer (2021) performed a systematic literature review to synthesize the definitions of mathematical creativity. According to this review, mathematical creativity in mathematics education is defined as "novel mathematical ideas or products, which are new to the person but may not necessarily be new to others" (Bicer, 2021, p. 3).

Historically, mathematical creativity was predominantly associated with expert mathematicians (Hadamard, 1945, in Sriraman, 2009) and was thus regarded as an absolute trait inherent to a select few. However, contemporary researchers advocated a broader view. Scholars like Leikin (2009) argue that mathematical creativity is not exclusive to experts but can also be identified in students, in their processes, or in the products of their work, suggesting a more relative interpretation of creativity. This expanded view acknowledges that creativity in the context of school mathematics is distinct from the creativity observed among professional mathematicians. Mathematical creativity in school is evaluated in relation to prior experiences and the performance of other students with a comparable educational background (Sriraman, 2005). Moreover, scholars like Gregoire (2016) argue that mathematical creativity is teachable. This approach underscores the importance of contextual factors in evaluating creativity, recognizing that it can manifest in various forms and levels across different educational landscapes.

Recent studies revealed that mathematical creativity is domain-specific, not general (e.g., Kattou et al., 2015). By combining thinking with creativity, flexibility, and various ideas, mathematics becomes more engaging (Boaler, 2016). Furthermore, creativity should be encouraged in the mathematics classroom because mathematical creativity impacts the quality of mathematical knowledge (e.g. Bicer et al., 2020; Tabach & Friedlander, 2013). Namely, the ability to solve mathematical problems through varied approaches and from multiple perspectives deepens students' understanding of mathematical concepts and fosters a more integrated knowledge, and connects mathematics with various mathematical domains.

Teaching approaches to foster mathematical creativity

Specific teaching approaches have been suggested to cultivate mathematical creativity in the classroom. One of the most advocated teaching approaches to foster mathematical creativity and divergent thinking the utilization of open-endedness in classrooms (Pehkonen, 1995; Klein & Leikin, 2020; Levenson et al., 2018). Open-ended or open tasks lend themselves to many solution paths and strategies (Pehkonen, 1995). Open tasks contrast a traditional task where one very specific method is deemed correct by the teacher. Open-mindedness also allows students to hear diverse and divergent thinking patterns from peers. Solving open mathematical tasks is indeed a creativity-directed activity because it stimulates and demands mental flexibility and affords multiple opportunities to produce new ideas (Klein & Leikin, 2020). New information is forged by combining mathematical structures, processes, strategies, and creative thinking (Vale et al., 2018). Open tasks can be seen as an umbrella class that includes problems of three major types based on the openness of the starting situation (open-start problems) and the openness of the goal situation (open-end problems) defined by the task (Pehkonen, 1995). Leikin (2018) classified open tasks into several categories open start, open end, and combined problems:

- Multiple Solution Tasks —These tasks have an open starting point. That is, there are a variety of possible solution methods to the problem, each resulting in the same solution.

- Multiple outcome tasks—Solving these tasks results in multiple answers. They are either open-ended or combined (open-ended and open-start).
- Investigation Tasks — These tasks can be both open-start and open-end tasks, as they can be approached in various ways (at the beginning) and can result in various discoveries (at the end).
- Sorting Tasks also combine open start and end when criteria for sorting mathematical objects are not provided. Students must invent sorting criteria, and various sorting criteria result in distinct sorting outcomes — categories of mathematical objects.
- Problem Posing Tasks - combine open-start and open-end tasks. Participants are required to formulate problem statements that are novel to the poser, based on a given set of conditions.

Open tasks require general and specific cognitive skills (Vale et al., 2018). The following section outlines various open tasks that promote (and allow assessment of) mathematical creativity in the classroom.

Fostering creativity using open tasks

Multiple solution tasks

Multiple Solution Tasks (MST) are open tasks that contain explicit requirements to be solved using different methods. Leikin (2009) used MSTs to introduce the operational definition of mathematical creativity in a school context. At the school level, mathematical creativity can be regarded as (mental) flexibility (1) to solve a particular problem with multiple methods, (2) to implement strategies from various mathematical domains, and (3) to produce insightful solutions to a problem. These indicators can be observable in the context of MST as students solve problems by 1) using different representations of mathematical objects, 2) using different definitions or theorems of mathematical concepts, and 3) using different properties of mathematical objects in different disciplines and in real life (Levav-Waynberg & Leikin, 2012). Leikin (2009) suggested a notion of solution spaces that enables researchers to examine the various aspects of problem-solving performance using MSTs. The individual solution space represents all sets of solutions generated by a person without the assistance of another person; a collective solution space represents solutions generated by a group of participants; and an expert solution space represents solutions produced by expert mathematicians (Levav-Waynberg & Leikin, 2012). Furthermore, expert solution spaces can be both conventional and unconventional. In contrast to unconventional solution spaces, conventional solution spaces are recommended by the curriculum, displayed in textbooks, and taught by teachers (Leikin, 2009).

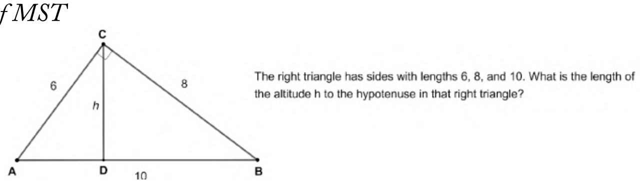
To assess mathematical creativity through MSTs, Leikin proposed a scoring scheme predicated on solution spaces. The scheme includes fluency as the number of appropriate solutions, flexibility as solution categories, and originality of solutions—judged by relative and absolute criteria. Originality is estimated by the uniqueness of the solution methods, with a method deemed original if it appears in 15% or fewer of the collected solutions. Details of the scoring scheme can be found in Leikin (2009).

As a measure of mathematical creativity, MSTs were used in various studies. MST served as a research instrument to assess geometry knowledge and mathematical creativity among high-ability and regular-ability students (Levav-Waynberg & Leikin, 2012), to examine the relationship between mathematical creativity, mathematical ability, and general giftedness (Leikin & Lev, 2013), or to investigate connections between creativity and mathematical knowledge among middle-school students (Tabach & Friedlander, 2013). Moreover, the MST examined the creative processes that arise when solving problems (Schindler & Lilienthal, 2020). Research on the effectiveness of MST in promoting mathematical creativity among school students has yielded encouraging results. Studies reported positive effects on the coherence of mathematical knowledge, abilities of problem-solving, and higher-order thinking skills (e.g., Levav-Waynberg & Leikin, 2012; Tabach & Friedlander, 2018).

The example of MST in Figure 1 was taken from Jukić Matić & Sliško (2024). The task can be found in various materials, like school textbooks, exam preparation manuals, and online student tutorials. However, those materials do not usually ask for multiple solution methods.

Figura 1

Example of MST



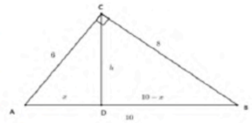
The study's authors converted it into MST, posing the requirement for multiple solution methods. The task can be solved with knowledge of secondary school mathematics, which makes it accessible to a wide range of secondary students, not just gifted or high-ability ones. The expert solution space of the above task consists of nine solution methods; eight solution methods were classified as conventional, while the ninth method was classified as non-conventional (Figure 2). Eight conventional methods within the collective solution space were identified in the study. The most used methods were the Pythagorean theorem and the Area of the right-angle triangle.

Figura 2

Expert solution space from Jukić Matić & Sliško (2024)

Expert solution methods

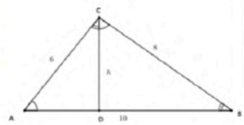
Solution method 1. Pythagorean Theorem
Applying Pythagorean Theorem twice on two smaller triangles ADC and DBC, the following system of equations is obtained:
$6^2 = x^2 + h^2$, $8^2 = (10 - x)^2 + h^2$.
Solving this system we obtain $x = 3.6$, and $h = 4.8$.



Solution method 2. Area of right-angle tringle
If we denote with a and b the length of legs, the area of right-angled triangle can be calculated as $P = \frac{a \cdot b}{2} = 24$. Then from general formula for triangle area $P = \frac{c \cdot v_c}{2} = \frac{c \cdot h}{2}$ follows $h = 4.8$.

Solution method 3. Heron's formula
Let us denote $a = 8$, $b = 6$, a $c = 10$. From Heron's formula $P = \sqrt{s(s-a)(s-b)(s-c)}$ the area P =24 is obtained. Then from the general formula for triangle area $P = \frac{c \cdot v_c}{2} = \frac{c \cdot h}{2}$ follows $h = 4.8$.

Solution method 4. Similarity of triangles
Triangles ADC and ABC have two angles with same sizes e.g. $\angle DAC$ is joint angle, and both triangles are right-angled ones. Thus, these triangles are similar according to the Angle-Angle Theorem. In the same way, we conclude that triangles DBC and ABC are also similar. Consequently, triangles ADC and DBC are similar. Using ratios of the corresponding side lengths, e.g. $h : 6 = 8 : 10$, one obtains $h = 4.8$.



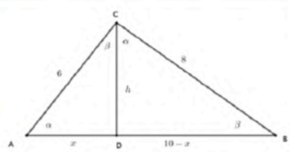
Solution method 5. Altitude geometric mean theorem (Theorem of Euclid)
Let us denote $p = |AD|$ *and* $q = |DB|$. Further let us denote $a = 8$, $b = 6$, and $c = 10$. From geometric mean theorem: $a = \sqrt{cq}$, $b = \sqrt{cp}$ i $h = \sqrt{pq}$, it follows $p = \frac{b^2}{c} = \frac{36}{10}$, $q = \frac{a^2}{c} = \frac{64}{10}$. Length of the altitude h is obtained as $h = \sqrt{pq} = \frac{6 \cdot 8}{10} = 4.8$.

Solution method 6. Trigonometric ratios
Determine angle sizes from trigonometric ratios. For instance, we determine α from $\sin\alpha = \frac{8}{10}$, $\cos\alpha = \frac{6}{10}$ or $\tan\alpha = \frac{8}{6}$. $\alpha = 53°7'$.
Using trigonometric ratios on smaller triangle ADC, the altitude h is obtained from $\sin 53°7' = \frac{h}{6}$ as $h = 4.8$.

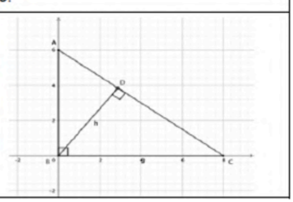
Solution method 7. Law of cosines and law of sines
Utilization of Laws of sines or cosines as generalizations of trigonometric ratios on any given triangle.

For instance, let's denote $a = 8$, $b = 6$, $c = 10$. Using the Law of cosines ($a^2 = b^2 + c^2 - 2bc\cos\alpha$) we determine $\cos\alpha$ and $\cos\beta$. Now from $\sin^2\alpha + \cos^2\alpha = 1$, $\sin^2\beta + \cos^2\beta = 1$, and the fact that $\alpha, \beta \in \langle 0°, 90° \rangle$, we determine $\sin\alpha$ and $\sin\beta$. The length of altitude h follows from the Law of sines $\frac{h}{\sin\alpha} = \frac{6}{\sin\beta} \Rightarrow h = 4.8$.

Solution method 8. Trigonometric ratios and same angles in different triangles
This solution method uses trigonometric ratios in smaller triangles and the fact that smaller triangles have angles of the same size.
For instance: let us find $\cos\alpha$ and $\cos\beta$.
First β: $\cos\beta = \frac{10-x}{8}$ and $\cos\beta = \frac{h}{6}$. From these two equations, we get $\frac{10-x}{8} = \frac{h}{6}$ or $10 - x = \frac{8h}{6}$ (*). Then α: $\cos\alpha = \frac{x}{6}$ and $\cos\alpha = \frac{h}{8}$. From these two equations, we get $\frac{x}{6} = \frac{h}{8}$ or $x = \frac{6h}{8}$ (**)
Substituting x from (**) into (*), we obtain: $10 - \frac{6h}{8} = \frac{8h}{6}$. The altitude follows as $h = 4.8$.



Solution method 9. Coordinate geometry
Place the triangle ABC into the coordinate system, with legs on the x and y axis. Now triangle vertices have coordinates $A(0,6)$, $B(0,0)$ and $C(8,0)$. Determine equation of line AC from coordinates of A and C: $y = \frac{-3}{4}x + 6$. Line BD is perpendicular to line AC: $y = \frac{4}{3}x$. Point D is the intersection of lines AC and BD. Coordinates of point D are (2.88, 3.84). Segment $\overline{BD}$ is the altitude of triangle ABC: $|BD| = \sqrt{2.88^2 + 3.84^2} = 4.8$.



Multiple outcome tasks

Levenson et al. (2018) examined mathematical creativity using open tasks with a requirement to generate as many answers as possible. Although Levenson et al. call these tasks open tasks, we will use the term Multiple

Outcome Tasks (MOT), relying on Leikin's (2018) categorization. To assess mathematical creativity using MOT, Molad et al. (2020) propose a model based on fluency, which refers to the number of correct but noncongruent answers; flexibility, which considers answer categories; and originality, which is determined by frequency and insight. Generally, an answer that is present in 15% or less of the solutions is considered original. Tasks like MOT offer better opportunities to promote flexibility in the school environment because a task intended for use in a specific environment should also be evaluated by analyzing the outcomes of its implementation in that environment (Levenson et al., 2018). Example 1 is an example of MOT taken from Molad et al. (2020). The students' solutions to this task included various convex and non-convex polygons, regular and irregular ones. The methods students used to create shapes with the required area were counting squares of grid paper or calculating the area of a particular shape. Some students generated almost 50 shapes, but researchers only considered some to be appropriate. The task itself has infinite solutions.

Example 1: Construct multiple polygons with an area of 15 square units.

Certain MOTs, such as the one presented above, facilitate the utilization of a wide array of problem-solving strategies and lead to the discovery of various solutions. This dual focus allows educators and researchers to assess both the products and processes of creativity. By analyzing the solutions that students generate, one can gauge the creativity of the outcomes—this reflects the product aspect of creativity. On the other hand, evaluating the different strategies students employ to arrive at these solutions offers insights into the creative process itself, similar to the approach with MSTs (Levenson et al., 2018). Klein and Leikin (2020) caution that while MSTs are inherently open-start tasks, MOTs are not always open-end tasks. However, to qualify as a task that fosters creativity, MOT must be open-ended (Bicer et al., 2020). Example 2 shows an example of MOT, which cannot necessarily be considered an open-ended task because it requires a complete set of outcomes (Leikin & Elgraby, 2022).

Example 2: Construct all possible rectangles for which the area is 120 square units, and the sides' lengths are whole numbers of units.

Problem posing

Problem posing is an open task, which can serve as a method to stimulate creativity, especially when there is an explicit requirement that the person posing the problem poses as many problems as possible for the given situation. According to Bevan and Capraro (2021), this approach not only leverages students' prior knowledge but also encourages the integration of real-world scenarios, promoting critical and adaptable thinking. However, not all problem-posing activities are creativity-directed activities. To understand that, let us examine approaches to problem-posing. Research on problem-posing reveals two theoretical frameworks: Stoyanova and Ellerton (1996) and Silver (1994). The first theory categorizes problem-posing into free, semi-structured, and structured categories. Structured problem-posing asks for reshaping a previously solved problem or changing the conditions or questions of given problems. In constrast, semi-structured problem-posing requires students to complete an open scenario using previous mathematical knowledge and experience. Free problem-posing involves students creating their problems based on given situations. Silver (1994) defines problem-posing as either restating or introducing new problems. Restatement of an existing problem entails describing the problem in a new way by changing the problem statement of a solved problem. While both theoretical frameworks are used by researchers, Stoyanova and Ellerton's (1996) approach has received more attention (e.g., Bonotto, 2013; Bonotto & Dal Santo, 2015; Van Harpen & Presmeg, 2013). Moreover, semi-structured or unstructured situations foster mathematical creativity more than structured ones because such situations stimulate student sensitivity to a problem (Bonotto & Dal Santo, 2015).

When posing problems, one can also assess components of mathematical creativity such as fluency, flexibility, and originality (Van Harpen & Sriraman, 2013). Fluency can refer to the number of posed problems, flexibility to the diversity of posed problems (e.g., in terms of different mathematical ideas or strategies to be applied), and originality to the rareness of the posed problems compared within the solution space of a peer group. If 10% or more of the participants in that group proposed the problem, it is considered original. Problem posing has other advantages. If teachers allow students to pose problems, they can gain valuable insight into students' mathematical viewpoints, their prior knowledge, and current understanding of the concepts used (Kiliç, 2017).

Figure 2 is an example of PP used in intervention among elementary school students in a study by Bicer et al. (2020).

Figura 3

Example of problem-posing activity

Make up as many problems as possible by using the menu shown below.
You and a friend go to lunch with $20. Set up your solutions.

Menu	
Pork Tenderloin	$ 8.99
French Dip	$ 7.47
Reuben	$ 6.50
Turkey Club	$ 5.49

The students who were assigned to the problem-posing group showed an increase in mathematical creativity compared with results before the intervention. Moreover, the study indicates that integrating problem-posing strategies into elementary mathematics classrooms (grades 3–5) can help students develop creative abilities.

Another approach to mathematical creativity

Prabhu and Czarnocha (2014) advocate for a more comprehensive approach to the study of mathematical creativity, stressing the importance of incorporating students from underprivileged communities and those with negative feelings toward the subject. They emphasize how crucial it is to create a stimulating environment and turn negative experiences into positive ones. Specifically, Prabhu (2016) found that affective barriers—rather than cognitive barriers—are the main obstacles that underprivileged communities encounter in mathematics classrooms. Students' self-perception of mathematical failure hindered them more than actual comprehension gaps. Additionally, Czarnocha (2014) emphasizes the significance of sudden understanding, or the 'Aha moment,' especially among marginalized students. He argues that this instant insight is a crucial part of creativity, which is frequently overlooked in earlier discussions. This concept aligns with Koestler's (1964, in Czarnocha, 2014) definition of creativity, highlighting the spontaneous nature of creative insights.

Koestler views creativity as a key element within a problem-solving context, defining it as moving from an initial situation to a desired goal. He focuses on the 'Aha moment' or bisociation, which he describes as a sudden cognitive leap that often leads to insights. This Aha moment requires what Koestler calls a bisociative frame. This mental framework connects two seemingly unrelated sets of ideas or experiences to uncover a hidden analogy, the core of creative insight. According to Czarnocha (2014), Koestler's idea of bisociation integrates insights from three different fields: humor, discovery, and art, identifying bisociation as the common thread that underpins the Aha moment across these fields. This idea is presented through a triptych that symbolizes the connection of humor with discovery and discovery with art:

comic comparison ↔ objective analogy ↔ poetic analogy

The first part of the triptych aims to awaken laughter and cheerfulness; the second leads to a deeper understanding; and the third introduces the world of wonder and surprise. After recognizing the humorous dimension and the core of the concept, it is necessary to delve into its deeper layers for a complete understanding and finally interpret it through the prism of art. Although the emotional and expressive tone changes within the triptych, the basic concept of bisociation is consistent across all domains. Czarnocha et al. (2018) point out that bisociation provides a solution to the classic division of creativity into specific and general, arguing that both aspects coexist at the base of every creative phenomenon: the universal principle of bisociation works in parallel with its specific application within a particular domain. Considering its form, the triptych also represents an open task. As an example of the triptych, we present one from the domain of statistics (Czarnocha et al., 2016):

trailblazer ↔ outlier ↔ originality
↔ sampling ↔
↔ probability ↔
↔ confidence interval ↔
↔ law of large numbers ↔

The given triptych illustrates the idea, and students should fill in the left and right sides e.g., probability.

Prabhu and Czarnocha (2014) point out that tasks in the form of a triptych allow students to identify connections between key mathematical concepts and actively participate in the learning process. This method is beneficial because it encourages all students to get involved and contribute, even those who face internal barriers, such as the belief that they cannot do it. Czarnocha (2021) contends that when analyzing Aha moments in mathematics, the focus should be on their ability to advance their understanding of specific mathematical ideas rather than their appearance. Analyzing Aha moments allows us to be understand better how they deepen students' conceptual knowledge and lead to a more comprehensive understanding of mathematics. Furthermore, Czarnocha proposes qualitative indicators for evaluating the depth of Aha moments based on the concept of bisociation. Mild bisociation involves only one conceptual connection or analogy. This includes the process in which discovering a hidden analogy involves elementary conceptions. Normal bisociation refers to cognition, in which several elementary concepts are coordinated to form a functional whole. A functional whole is defined as creating an internal schema that leads to a solution or a higher level of understanding. Strong bisociation contains at least two steps or cycles in the development of understanding. In this way, one can assess the depth of the Aha moment during and immediately after its duration. Czarnocha further argues that by focusing assessment on change in student understanding, external constraints on what counts as creative are eliminated, and creativity can be assessed at whatever level it occurs.

It seems that Czarnocha has created an alternative assessment of creativity that is not based on flexibility and fluency. Moving away from fluency and flexibility, on the one hand, can be convenient and useful, because Leikin and Lev (2013) indicated a decrease in originality during an experiment based on Torrance's definition, which included flexibility, fluency, and originality.

Classroom environment

As noted in the previous section, the learning environment significantly impacts mathematical creativity. Numerous studies have examined which factors, apart from suitable tasks, impact the development of creativity. One is the interaction among students in the classroom (Molad et al., 2020; Levenson & Molad, 2022). In their study, Molad et al. (2020) investigated postsecondary students' mathematical creativity through individual and collaborative work. The researchers discovered that students who had engaged in group work exhibited higher levels of fluency and flexibility in their work than those who had performed individual work. Additionally, group activities helped students learn new mathematical concepts that they would later use independently. Furthermore, students had the chance to learn about different ideas and ways to solve problems, to think critically, and to expand their knowledge of math topics. Levenson and Molad (2022) detected that collaborative groups are capable of producing unexpected, novel solutions. They found that flexibility increases with greater collaboration among group members; likewise, collective fluency improves when a diverse group of students collaborates in a manner that allows everyone to contribute.

Other factors are also influential in the school environment. Cho (2007) points out that students ought to have the autonomy to engage in creative activities without fear of the consequences of failure; teachers should encourage unanticipated responses and honor original ideas. Similarly, Kaufman & Sternberg (2006) argue that students should be encouraged to express their creative ideas, notwithstanding the potential for dissent from other students. Another element that promotes the development of mathematical creativity is parental and academic support. Teachers and parents should create an atmosphere free from tension and anxiety wherein students are encouraged to commit errors and encounter setbacks (Pham & Cho, 2018).

Classroom environment

This paper presents various open tasks appropriate for promoting mathematical creativity in a school environment. Open tasks were selected as the focus over other approaches available, given their partial inclusion in school textbooks (e.g., Bingolbali, 2020; Cai & Jiang, 2017; Yang et al., 2017). Namely, research shows that the tasks used in mathematics lessons are similar to those in the textbooks (Schmidt, 2012). Therefore, teachers' ability to integrate creativity-directed tasks into their mathematics instruction partly depends on the textbooks they use. Some studies found that as students move up the grade levels, the number of creativity-directed activities in general and open tasks in particular decreases (Bicer et al., 2021; Hadar & Tirosh, 2019), giving the impression that creativity is the domain of younger students. When it comes to encouraging mathematical creativity, teachers have the final say in selecting and implementing activities within the classroom (Freiman, 2009). Yet, challenges persist, including teachers' resistance to open tasks due to a lack of recognition of their benefits, difficulties in assessment, and the need for flexibility in posing and implementing such tasks (Klein & Leikin, 2020; Mihajlović & Dejić, 2015). Additionally, the emphasis on standardized assessment further inhibits the incorporation of creativity-directed tasks (Brookhart, 2013; Long et al., 2022).

A new pedagogy is needed to cultivate mathematical creativity effectively: one that fosters an open classroom atmosphere, integrates creativity-directed activities, and draws upon interdisciplinary ideas and real-world experiences (Schoevers et al., 2019). This approach requires a paradigm shift in teacher education and educational policy. First and foremost, education policies must be revised. Policies should encourage the inclusion of creativity-focused tasks at all grade levels, challenging the myth that creativity is only for younger students. Policymakers should advocate for curriculum standards and assessment methods that promote and measure creativity. Furthermore, professional development programs should provide teachers with the knowledge and skills they need to recognize the value of mathematical creativity, design and implement creativity-directed tasks, and effectively assess creative work. Training should also focus on increasing teachers' confidence in dealing with the complexities and uncertainties of such tasks. This should include creating a classroom environment that promotes risk-taking, collaboration, and open discussion.

References

Bevan, D., & Capraro, M. M. (2021). Posing creative problems: a study of elementary students' mathematics understanding. *International Electronic Journal of Mathematics Education, 16*(3), em0654. https://doi.org/10.29333/iejme/11109

Bicer, A. (2021). A systematic literature review: Discipline-specific and general instructional practices fostering the mathematical creativity of students. *International Journal of Education in Mathematics, Science, and Technology, 9*(2), 252–281. https://doi.org/10.46328/ijemst.1254

Bicer, A., Lee, Y., Perihan, C., Capraro, M. M., & Capraro, R. M. (2020). Considering mathematical creative self-efficacy with problem posing as a measure of mathematical creativity. *Educational Studies in Mathematics, 105*(3), 457–485. https://doi.org/10.1007/s10649-020-09995-8

Bicer, A. Marquez, A., Valesca, Colindres, K. V. M., Schanke, A. A., Castellon, L. B., Audette, L. M, Perihan, C. & Lee, Y. (2021). Investigating creativity-directed tasks in middle school mathematics curricula. *Thinking Skills and Creativity, 40*, 100823 https://doi.org/10.1016/j.tsc.2021.100823

Bingolbali, E. (2020). An analysis of questions with multiple solution methods and multiple outcomes in mathematics textbooks, *International Journal of Mathematical Education in Science and Technology, 51*(5), 669–687. https://doi.org/10.1080/0020739X.2019.1606949

Boaler, J. (2016). *Mathematical mindsets: Unleashing student's potential through creative math, inspiring messages, and innovative teaching.* JB Jossey-Bass.

Bonotto, C. (2013). Artifacts as sources for problem-posing activities. *Educational Studies in Mathematics, 83*(1), 37–55. https://doi.org/10.1007/978-1-4614-6258-3_5

Bonotto, C., & Dal Santo, L. (2015). On the relationship between problem posing, problem solving, and creativity in the primary school. In F. M. Singer, N. Ellerton, & J. Cai (Eds.), *Mathematical problem posing: From research to effective practice* (pp. 103–123). Springer.

Brookhart, S. M. (2013). The use of teacher judgement for summative assessment in the USA. *Assessment in Education: Principles, Policy & Practice, 20*(1), 69–90, https://doi.org/10.1080/0969594X.2012.703170

Cai, J., & Jiang, C. (2017). An analysis of problem-posing tasks in Chinese and US elementary mathematics textbooks. *International Journal of Science and Mathematics Education, 15*(8), 1521–1540. https://doi.org/10.1007/s10763-016-9758-2

Cho, S. (2007). Nurturing creative problem solving ability of the gifted in Confucian society. *Journal of Gifted/Talented Education, 17*(2), 392–411.

Czarnocha, B. (2014). O kulturi kreativnosti u matematičkom obrazovanju [Sobre la cultura de la creatividad en la educación matemática]. *Inovacije u nastavi - časopis za savremenu nastavu , 27*(3), 31–45. https://doi.org/10.5937/inovacije1403031C

Czarnocha, B. (2021). Assessment of the depth of knowledge acquired during an Aha! moment insight. In B. Czarnocha, & W. Baker (Eds.), *Creativity of an Aha! moment and mathematics education* (pp. 110–138). Brill Publisher.

Czarnocha, B., Baker, W., & Dias, O. (2018). Creativity research in mathematics education simplified: using the concept of bisociation as Ockham's razor. In P. Ernest (Ed.), *The philosophy of mathematics education today. ICME-13 Monographs* (pp. 321–332). Springer. https://doi.org/10.1007/978-3-319-77760-3_20

Czarnocha, B., Baker, W., Dias, O., & Prabhu, V. (2016). *The creative enterprise of mathematics teaching research*. Sense Publishers.

Elgrably, H., & Leikin, R. (2021). Creativity as a function of problem solving expertise: Posing new problems through investigations. *ZDM, 53*(4), 891–904. https://doi.org/10.1007/s11858-021-01228-3

Freiman, V. (2009). Mathematical enrichment: Problem-of-the-week model. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 367–382). Sense Publishers.

Grégoire, J. (2016). Understanding creativity in mathematics for improving mathematical education, *Journal of Cognitive Education and Psychology, 15*(1), str. 24–36. https://doi.org/10.1891/1945-8959.15.1.24

Guilford, J. P. (1950). *Creativity. The American Psychologist, 5*(9), 444–454.

Hadar, L. L., & Tirosh, M. (2019). Creative thinking in mathematics curriculum: An analytic framework. *Thinking Skills and Creativity, 33*, 100585. https://doi.org/10.1016/j.tsc.2019.100585

Jukić Matić, Lj., & Sliško, J. (2024). An empirical study of mathematical creativity and students' opinions on multiple solution tasks. I*nternational Journal of Mathematical Education in Science and Technology, 55*(9), 2170–2190. https://doi.org/10.1080/0020739X.2022.2129496

Kattou, M., Christou, C., & Pitta-Pantazi, D. (2015). Mathematical creativity or general creativity? In K. Kaiser & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 1016–1023). Charles University in Prague, Faculty of Education and ERME.

Kaufman, J. C., & Sternberg, R. J. (Eds). (2006). *The international handbook of creativity.* Cambridge University Pres

Kiliç, C. (2017). A new problem-posing approach based on problem-solving strategy: Analyzing pre-service primary school teachers' performance. *Educational Sciences: Theory and Practice, 17*(3), 771–789.

Klein, S. & Leikin, R. (2020). Opening mathematical problems for posing open mathematical tasks: what do teachers do and feel? *Educational Studies in Mathematics, 105*(3), 349–365. https://doi.org/10.1007/s10649-020-09983-y

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129–135). Sense Publishers.

Leikin, R. (2018). Openness and constraints associated with creativity-directed activities in mathematics for all students. In N. Amado, S. Carreira, & K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving* (pp. 387–397). Springer.

Leikin, R. & Elgrably, H. (2022). Strategy creativity and outcome creativity when solving open tasks: focusing on problem posing through investigations. *ZDM, 54*(1), 35–49. https://doi.org/10.1007/s11858-021-01319-1

Leikin, R., & Lev, M. (2013). Mathematical creativity in generally gifted and mathematically excelling adolescents: What makes the difference? *ZDM, 45*(2), 183–197. https://doi.org/10.1007/s11858-012-0460-8

Leikin, R., & Sriraman, B. (2016). Introduction to Interdisciplinary Perspectives to Creativity and Giftedness. In R. Leikin, & B. Sriraman (Eds.), *Creativity and giftedness: Interdisciplinary perspectives from mathematics and beyond* (pp. 1–3). Springer. https://doi.org/10.1007/978-3-319-38840-3

Levav-Waynberg, A., & Leikin, R. (2012). The role of multiple solution tasks in developing knowledge and creativity in geometry. *The Journal of Mathematical Behavior, 3*1(1), 73–90. https://doi.org/10.1016/j.jmathb.2011.11.001

Levenson, E. S., & Molad, O. (2022). Analyzing collective mathematical creativity among post high-school students working in small groups. *ZDM, 54*(1), 193–209. https://doi.org/10.1007/s11858-021-01321-7

Levenson, E., Swisa, R., & Tabach, M. (2018). Evaluating the potential of tasks to occasion mathematical creativity: Definitions and measurements. *Research in Mathematics Education, 20*(3), 273–294. https://doi.org/10.1080/14794802.2018.1450777

Long, H., Kerr, B.; Emler, T. E., & Birdnow, M. (2022). A critical review of assessments of creativity in education. *Review of Research in Education, 46*(1), 288323. https://doi.org/10.3102/0091732X221084326

Mihajlović, A., & Dejić, M. (2015). Using open-ended problems and problem posing activities in elementary mathematics classrooms. In F. M. Singer, F. Toader, & C. Voica (Eds.), *Proceedings of the 9th Mathematical Creativity and Giftedness international conference* (pp. 34–41). Nanyang Technological University.

Molad, O., Levenson, E. S., & Levy, S. (2020). Individual and group mathematical creativity among post–high school students. *Educational Studies in Mathematics, 104*(2), 201–220. https://doi.org/10.1007/s10649-020-09952-5

Organization for Economic Co-operation and Development [OECD]. (2019). *OECD future of education and skills 2030: OECD learning compass 2030.* OECD.

Pehkonen, E. (1995). Introduction: Use of open-ended problems. *ZDM, 27*(2), 55–57.

Pham, H. L. & Cho, S. (2018). Nurturing mathematical creativity in schools. *Turkish Journal of Giftedness and Education, 8*(1), 65–82.

Prabhu, V. (2016). The creative learning environment. In B. Czarnocha, W. Baker, O. Dias & V. Prabhu (Eds.), *The creative enterprise of mathematics teaching research* (pp. 107–126). Sense Publishers.

Prabhu, V., & Czarnocha, B. (2014). Democratizing mathematical creativity through Koestler's bisociation theory. In S. Oesterle, P. Liljedahl, C. Nicole, & D. Allan (Eds.). (2014). *Proceedings of the joint meeting of PME 38 and PME-NA 36* (Vol. 3) (pp. 1–8). PME.

Schindler, M., & Lilienthal, A. (2020). Students' creative process in mathematics: Insights from eye-tracking-stimulated recall interview on students' work on multiple solution tasks. *International Journal of Science and Mathematics Education, 18*, 1–22.

Schmidt, W. (2012). Measuring content through textbooks: The cumulative effect of middle-school tracking. In G. Gueudet, B. Pepin & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 143–160). Springer.

Schoevers, E. M., Leseman, P. P. M., Slot, E. M., Bakker, A., Keijzer, R., & Kroesbergen, E. H. (2019). Promoting pupils' creative thinking in primary school mathematics: A case study. *Thinking Skills and Creativity*. *31*, 323–334. https://doi.org/10.1016/j.tsc.2019.02.003

Schukajlow, S., & Krug, A. (2014). Do multiple solutions matter? Prompting multiple solutions, interest, competence, and autonomy. *Journal for Research in Mathematics Education, 45*(4), 497–533.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM, 29*(3), 75–80. https://doi.org/10.1007/s11858-997-0003-x

Sriraman, B. (2005). Are giftedness & creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *The Journal of Secondary Gifted Education, 17*, 20–36. https://doi.org/10.4219/jsge-2005-389

Sriraman, B. (2009). The characteristics of mathematical creativity. *ZDM, 41*(1–2), 13–27. http://doi.org/10.1007/s11858-008-0114-z

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing. In P. Clarkson (Ed.), *Technology in mathematics education* (pp. 518–525). Mathematics Education Research Group of Australasia.

Tabach, M., & Friedlander, A. (2013). School mathematics and creativity at the elementary and middle grade level: How are they related? *ZDM, 45*(2), 227–238.

Tabach, M., & Friedlander, A. (2018). Instances of promoting creativity with procedural tasks. In M.F. Singer (Ed.), *Mathematical creativity and mathematical giftedness. Enhancing creative capacities in mathematically promising students* (pp. 83–114). Springer.

Tabach, M. & Levenson, E. (2018). Solving a task with infinitely many solutions: Convergent and divergent thinking in mathematical creativity. In N. Amado, S. Carreira, & K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving: a focus on technology, creativity and affect* (pp. 219–242). Springer.

Vale, I., Pimentel, T., & Barbosa, A. (2018). The power of seeing in problem solving and creativity: An issue under discussion. In N. Amado, S. Carreira, & K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving* (pp. 243–272). Springer.

Van Harpen, X., & Presmeg, N. C. (2013). An investigation of relationships between students' mathematical problem-posing abilities and their mathematical content knowledge. *Educational Studies in Mathematics, 83*(1), 117–132. https://doi.org/10.1007/s10649-012-9456-0

Van Harpen, X. Y., & Sriraman, B. (2013). Creativity and mathematical problem posing: An analysis of high school students' mathematical problem posing in China and the USA. *Educational Studies in Mathematics, 82*(2), 201–221. https://doi.org/10.1007/s10649-012-9419-5

Yang, D. C., Tseng, Y. K., & Wang, T. L. (2017). A comparison of geometry problems in middle-grade mathematics textbooks from Taiwan, Singapore, Finland, and the United States. *EURASIA Journal of Mathematics, Science and Technology Education, 13*(7), 2841–2857. https://doi.org/10.12973/eurasia.2017.00721a