

Workshop 3 – Reporting implementation projects

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Despite persistent reform efforts in mathematics education, the challenge remains: how to successfully scale innovations from local contexts to broader, system-wide implementation. While implementation research is on the rise in mathematics education, the research base in the area remains fragmented. We observe a movement toward consensus on key concepts, but reporting standards remain inadequate. Hill et al. (2024) have proposed guidelines on what needs to be reported, which can serve as inspiration. In my opinion, Hill et al.'s guidelines are perhaps too detailed to be discussed in full within the framework of a research article.

As an alternative, I will propose here that reporting on implementation projects should be based on three mutually interdependent relationships: 1) the interplay between teachers and innovation; 2) the boundary between the project and the school organization; and 3) the mutual influences between the school and the external environment. I believe that reporting that can serve as a basis for field development should describe two scenarios for each of the three dimensions mentioned above. One scenario describes

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the conditions for planning and implementation. The other scenario describes the deviations from the planned conditions that occurred during implementation.

As a case in point, I will describe the planning and design of the Algebra Monday project.

The Algebra Monday project

The Algebra Monday project connects teacher teams across all school levels to jointly design, test, and refine a coherent approach to algebra teaching from primary school through upper secondary education. The work is grounded in a theory of how conceptual understanding develops over time (Helenius & Ahl, 2025; Vergnaud, 2009) and informed by implementation research and scholarship on scaling research-based innovations (Aguilar et al., 2023). A key resource for the project is a research-informed framework for algebra and early algebra (Helenius & Ahl, 2024), developed by synthesizing and building on prior research.

Mathematics education research has long shown that students typically progress from concrete representations to more abstract, symbolic reasoning (Suh et al., 2020). For younger learners, however, the symbolic systems of formal mathematics are not immediately transparent. Early mathematical activity, therefore, should begin in meaningful, familiar situations that can be gradually internalized and re-expressed in increasingly abstract forms.

Even though beginning with contextual and concrete representations is well justified, many students never fully transition beyond this level during their schooling. This pattern, shaped in part by long-standing pedagogical traditions associated with figures such as Johann Heinrich Pestalozzi, can restrict students' opportunities to engage in algebraic thinking. International assessments, including TIMSS (Grønmo, 2018), indicate that algebra is often underemphasized, not only in Nordic settings but also across many English-speaking systems. In Sweden, the issue is particularly pronounced: research repeatedly points to enduring shortcomings in algebra instruction, and recent textbook analyses (Helenius & Ahl, 2024) identify structural discontinuities in how algebraic reasoning is expected to develop in classroom practice.

Against this background, there is a strong need for a developmentally coherent strategy for algebra teaching across the Swedish school system, one that may also contribute insights to international research. The term Algebra Monday derives from an instructional model developed by the principal investigator, Linda Marie Ahl, during her work in adult education within the Swedish Prison and Probation Service. The label functions as an organizing principle, emphasizing that algebra should be a continuous

thread across grades and mathematical topics rather than a discrete unit. While the Algebra Monday implementation in grades 0–3 (ages 6–10) follows a different design, the focus here is on the setup used from grade 4 onward.

The Algebra Monday innovation comprises two tightly linked elements: (1) a teaching model and (2) a professional development model. Together, these components are intended to strengthen algebra instruction and support sustained implementation across diverse educational contexts.

The teaching model

Algebra Monday is a recurring teaching model in which students encounter a small number of short, carefully selected algebra tasks at the beginning of each week. Its primary purpose is not to introduce new content, but to develop students' fluency, conceptual understanding, and ability to recognize mathematical structures through regular practice. Because the activity recurs in a familiar format, it establishes a clear classroom routine: students know what is expected and can direct their attention towards the mathematics. The tasks can first be explored individually and then discussed in pairs or as a whole class. The teacher's role is to highlight different lines of reasoning, connect ideas, and ask questions such as "Why does this method work?", "Which factors are being canceled?", and "Can the expression be written in another way?"

The board shown in Figure 1 illustrates how five apparently simple and seemingly unrelated tasks can form a coherent mathematical whole. For grades 4–9, all lessons are based on sets of tasks like this, and students can start working with them in any order. In the first task, the central idea is to identify the common factors and explain why they can be canceled, using that for any $a \neq 0$, $\frac{a}{a} = 1$. The second task presents the same structure in a more abstract form. Here too, the quotient is 1. Comparing the two tasks can help students recognize that rules for algebraic expressions are based on the same principles as rules for numerical calculations.

The third task is designed to introduce the substitution view of equality and is something of an oddity among these five tasks, as it is the only one that does not concern fractions. We always try to mix tasks in this way to intentionally move away from the idea that every lesson is focused on a given mathematical content.

In the fourth task, the focus is again on allowing a discussion of the general rule that $\frac{a}{a} = 1$ and that extending or reducing fractions amounts only to finding common factors and using $\frac{a}{a} = 1$. A fraction can be reduced if and only if the numerator and denominator have common non-trivial

factors. The last task also concerns factorization, in the sense that a sum is rewritten as a product using the distributive property.

The examples illustrate a central principle of Algebra Monday: progression does not necessarily require longer or more complicated calculations. Challenge can instead arise through generalization, explanation, comparison, and the identification of relationships. The board therefore functions both as a space for practice and as a starting point for mathematical discussion, in which students' different solution strategies become a shared resource for learning.

Figure 1

Five Algebra Monday tasks

$$1. \frac{\sqrt{3}}{2} \cdot \frac{2}{\sqrt{3}} =$$

$$2. \frac{a^5}{a^5} =$$

$$3. \frac{1}{3}x = 1 \quad \frac{1}{3}x + 4 = _$$

4. Which fractions can be simplified and why?

$$\frac{24}{36} \quad \frac{23}{36} \quad \frac{24}{37} \quad \frac{25}{36}$$

$$5. \frac{2025 + 2025 + 2025}{2025 + 2025 + 2025 + 2025 + 2025} =$$

The professional development model

The professional development component comprises weekly 90-minute online sessions. During the pilot stage, these meetings are facilitated by researchers, who contribute both theoretical perspectives and practical experience in a structured format for small groups of mathematics teachers (Sjölund et al., 2022). Ahead of each meeting, teachers prepare by selecting, scanning, and bringing examples of student work, both responses that demonstrate strong algebraic thinking and samples that illustrate recurring difficulties or misconceptions.

Holding the sessions digitally makes participation feasible for teachers from multiple schools across the municipality. Responsibility for leading the meetings rotates: one teacher serves as moderator each week. The moderator guides the conversation by prompting participants to share reflections on recent classroom instruction. Student work is presented through screen sharing, with the teacher who contributed the samples explaining the task context and commenting on students' solutions. Colleagues indicate when they would like to respond, pose questions, or highlight issues. When questions arise regarding students' reasoning, symbolism, or mathematical justification, the moderator may invite the researcher to clarify points or contribute relevant expertise.

Overall, the model is designed to sustain inquiry-oriented professional dialogue anchored in objective classroom evidence. By centering discussion

on student work and distributing leadership through rotation, it promotes active participation and shared responsibility. Throughout the process, the researcher remains available as an ongoing resource, providing deeper instructional insight while keeping the work grounded in the project's theoretical foundations.

The relationship between teachers and innovation

The first dimension, the relationship between *teachers and innovation*, highlights that successful implementation hinges on teachers' engagement, professional judgment, and alignment with their existing knowledge and instructional routines. Teachers are not merely recipients of reform but active agents whose uptake is influenced by their perceptions of the innovation's value and feasibility. Reforms that challenge entrenched routines or demand new instructional schemes require time, support, and opportunities for collaborative reflection. Conversely, innovations that focus on curriculum content, sequencing, and alignment with teachers' existing pedagogical practices tend to be more readily implemented, even at scale.

What to report on the relationship between teachers and innovation

- Program content and determinants
- Duration
- PD organization
- Mandatory or voluntary Participation

The boundary between the project and the school organization

The second dimension concerns the *boundary between the project and the school organization*. As implementation efforts grow in scope, the relationship between external project teams and internal school structures must be carefully managed. Small-scale projects often rely on close, informal collaboration between researchers and teachers, whereas larger-scale efforts require more formalized and embedded coordination systems. Establishing stakeholder scaffolding within schools, hybrid groups that facilitate communication and support, can help shift ownership of the reform from external actors to internal stakeholders. Long-term sustainability depends on this gradual internalization, whereby schools assume increasing responsibility for sustaining the reform. To plan and evaluate implementation effectively, the innovation's complexity must inform the project's organizational design. Innovations that require profound shifts in teaching practice necessitate more complex, decision-capable project structures that integrate all levels of school governance—from policymakers and administrators to teachers. This alignment of roles and responsibilities is crucial for securing the necessary support structures

and for ensuring that responsibility for outcomes is properly distributed and understood. The larger the project, the more areas of responsibility of different stakeholders must be regulated within the project organization. Everything outside the project organization is considered the external environment in the three-dimensional model.

What to report on the boundary between the project and the school organization

- Which stakeholders have accepted which areas of responsibility within the project organization, and what are the arguments for the responsibility assigned to each group of stakeholders?

The mutual influences between schools and the external environment

The third dimension addresses the *mutual influences between schools and the external environment*. Implementation does not occur in isolation but is shaped by external policy, accountability systems, and public discourse. Coherence between the innovation and surrounding structures, curricula, assessments, and political priorities is critical. Misalignment, particularly with assessment regimes, can hinder uptake, while strong alignment can reinforce reform efforts and legitimize their place within the educational system. Moreover, schools themselves can influence the broader environment by contributing to professional discourse and shaping future policy directions.

What to report on the mutual influences between schools and the external environment

- How does the innovation relate to national curricula?
- How does the innovation relate to the content of high-stakes tests such as national exams?
- How do guardians of students react to the content of the innovation?

Ultimately, the three relational implementation dimensions described above offer a dynamic systems perspective for reporting on implementation projects, for declaring the planned project, and for making deviations from the planned project visible.

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References

- Aguilar, M. S., Ahl, L. M., Jankvist, U. T., & Helenius, O. (2023). Towards characterization of scale and scaling in implementation research within mathematics education. *Implementation and Replication Studies in Mathematics Education*, 3(1), 99–122. <https://doi.org/10.1163/26670127-bja10012>
- Grønmo, L. S. (2018). The role of algebra in school mathematics. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt, & B. Xu (Eds.), *Invited lectures from the 13th International Congress on Mathematical Education* (pp. 175–193). Springer. https://doi.org/10.1007/978-3-319-72170-5_11
- Helenius, O., & Ahl, L. M. (2024). A framework for analyzing long-term early algebra progression in textbook series. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.), *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 9–16). IGPME.
- Helenius, O., & Ahl, L. M. (2025). Conceptual fields and conceptual polysemy of mathematical objects with applications to the multiplicative conceptual field. *Hiroshima Journal of Mathematics Education*, 18, 51–69. <https://doi.org/10.24529/hjme.1804>
- Hill, H. C., Lynch, K., & Polanin, J. R. (2024). *Structured reporting guidelines for classroom intervention research* (EdWorkingPaper No. 24-1022). Annenberg Institute for School Reform at Brown University. <https://eric.ed.gov/?id=ED660203>
- Sjölund, S., Lindvall, J., Larsson, M., & Ryve, A. (2022). Using research to inform practice through research-practice partnerships: A systematic literature review. *Review of Education*, 10(1), Article e3337. <https://doi.org/10.1002/rev3.3337>
- Suh, S., Lee, M., & Law, E. (2020). How do we design for concreteness fading? Survey, general framework, and design dimensions. In *Proceedings of the Interaction Design and Children Conference* (pp. 581–588). Association for Computing Machinery. <https://doi.org/10.1145/3392063.3394413>
- Vergnaud, G. (2009). The theory of conceptual fields. *Human Development*, 52(2), 83–94. <https://doi.org/10.1159/000202727>

