

Algebra Monday: Designing and scaling theory-informed algebra instruction in Sweden

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Problem addressed

In mathematics education, it is widely recognized that learning progresses from concrete to abstract forms of reasoning (Suh et al., 2020). Concrete examples are essential in early mathematics learning because formal symbolic systems and reasoning are not yet accessible to students. To become meaningful, mathematical symbols and their relationships need to be anchored in experiences familiar to students, which can later be extended into abstract contexts. At the same time, research shows that many students remain at the concrete and illustrative stage of learning, a tradition that can be traced to the work of the 19th-century Swiss educator Johann Heinrich Pestalozzi. For these students, the transition to more symbolic and algebraic reasoning is not achieved. International surveys such as TIMSS indicate that algebra receives relatively limited emphasis in mathematics education, both in the Nordic countries and in English-speaking contexts (Grønmo, 2018). While this challenge is not unique to Sweden, it is particularly pronounced there, where algebra has been identified as a specific area of weakness, a trend confirmed in recent analyses of teaching materials (Helenius & Ahl, 2024).

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Theoretical foundations

The project *From Pestalozzi to Algebra Monday* builds on three theoretical frameworks. The first is Gérard Vergnaud's theory of conceptual fields (Vergnaud, 2009), which provides tools for planning mathematics instruction over the medium and long term. According to this theory, a concept is structured as a triplet $C = (S, I, R)$ where S refers to the classes of situations in which the concept is meaningful, I refers to the operational invariants that govern reasoning across those situations, and R refers to the representations used (Vergnaud, 1998). Our interpretation of Vergnaud emphasizes that invariants can be introduced only through classes of situations, iconic–schematic models, or as relations in symbol systems. This framework enables us to identify potential challenges for learners and develop effective long-term teaching strategies. For instance, while visual circular fraction models are widely used, research indicates that they may constrain mathematical thinking (Ahl & Helenius, 2024). Similar patterns have been documented for other core concepts (Ahl, 2016; Ahl & Helenius, 2021). The project thus seeks to validate the usefulness of conceptual field theory for structuring mathematics teaching.

The second framework concerns the conceptualization of school algebra. While traditionally associated with manipulating symbols and solving equations, recent research broadens the view of algebra as a cross-grade domain (Kieran, 2018). Our analysis identifies four perspectives: (1) algebra as structure (generalizing arithmetic relations), (2) algebra as operations on non-numeric symbols (classical equation solving), (3) algebra as functional thinking (expressing relationships such as $s(t) = 100t$), and (4) algebra as geometric patterns (repeating or growing patterns) (Helenius & Ahl, 2024).

The third framework is Design-Based Implementation Research (McKenney & Reeves, 2021), which guides the overall methodological approach of the project.

The implemented innovation

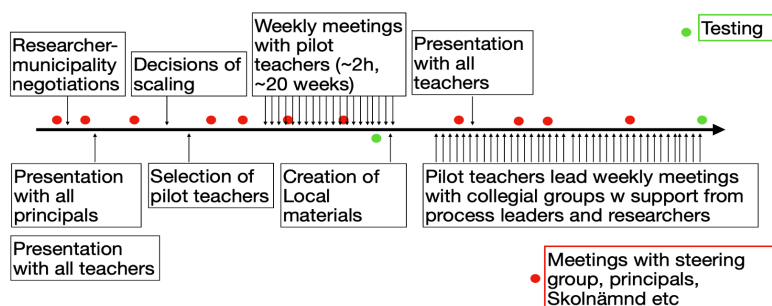
The innovation lies in dedicating one lesson per week, across all grades, to a specific instructional approach where the central focus is on the mathematical arguments that justify why particular operations are permissible. The teaching materials for these lessons, for grades 4–10, have been developed primarily by Linda Marie Ahl. During these lessons, students are presented with five interleaved tasks without any initial explanatory instruction from the teacher. The materials for grades K–3 have been developed primarily by Ola Helenius and follow an embodied cognition-oriented approach, with a systematic focus on symbol-system

representations. To support teachers in implementing Algebra Monday, weekly digital meetings provide an overview of the lesson content and highlight the algebraic reasoning targeted by the tasks. For the remainder of the week, teachers retain full autonomy to plan and deliver their mathematics instruction as they see fit.

Securing time for weekly meetings at the municipal level is often difficult, and teachers must also feel supported by their school leaders. Therefore, the project includes a dedicated organizational structure as a core part of the innovation (see Figure 1).

Figure 1

The temporal scaffolding project organization for working with municipalities



Hypothesis of potential importance

We hypothesize that Algebra Monday will enhance students' mathematical knowledge, as evidenced by improvements on national assessments and project-specific evaluation tools, while also contributing to international discussions on scalable, theory-informed approaches to strengthening algebra learning.

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