

From design to implementation for a Geogebra-mediated variational introduction to calculus

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What problem is addressed?

Upper-secondary school students frequently encounter functions and introductory calculus as a set of procedural routines. This procedural emphasis draws attention to static manipulations and suppresses covariational reasoning. Classroom practices often amplify this tendency, prioritising procedures over meaning-making, which limits students' access to conceptual understandings of core calculus ideas (e.g., Maher et al., 2022). Similar tensions have been documented in university calculus, where algebraic techniques tend to dominate over modelling and variational perspectives, even when technological tools such as GeoGebra are used in optimisation problems (Martínez-Miraval et al., 2025). In prior design-based

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work with 17-year-old students in Uruguay, the first author developed and trialled a GeoGebra-mediated sequence fostering covariational reasoning (Tejera, 2021). To build on that work, we now aim to derive a set of humble implementation principles (Bach et al., 2025). These principles will specify what must be preserved, what can be adapted, and what supports teachers will need to enact the sequence in their own settings.

How is it addressed?

We follow a design-to-implementation stance (Bach et al., 2025), in which design-research insights are made usable for wider implementation by articulating an explicit theory of change (ToC) and using it to guide analysis and the formulation of humble implementation principles. In this stance, a ToC is a structured account of how an innovation can be taken up, adapted, and sustained in real settings, with both conceptual and empirical components.

On the learning side, our ToC draws on Duval's (2017) theory of semiotic registers, Caballero-Pérez and Cantoral's (2013) characterisation of variational thinking and language, and Carlson et al.'s (2002) levels of covariational reasoning. The GeoGebra-mediated sequence is organised into variational task families intended to prompt students to work across algebraic, graphical, and verbal representations of change. Duval's distinction between treatment and conversion underpins the representational moves that tasks aim to elicit, while Caballero-Pérez and Cantoral's work guides situations and prompts that foreground variational aspects of phenomena. Carlson et al.'s framework provides an analytic lens to characterise students' covariational reasoning. Our hypothesis is that carefully sequenced variational tasks, enacted through teacher orchestration that makes cross-register links and variational language explicit, support students in developing richer, multi-register arguments about how quantities co-vary.

On the implementation side, we conceptualise the innovation as an ecological disruption (Koichu et al., 2021) that reconfigures classroom practices, technological infrastructures, and assessment priorities. To understand and support its uptake, we draw on Rogers' (2003) diffusion of innovations, focusing on how teachers and schools may perceive the innovation's attributes: its relative advantage (e.g., potential to deepen conceptual understandings of functions and calculus), compatibility (with curricula, institutional constraints, and current tool use), complexity (demands of GeoGebra Classroom and orchestration), trialability (possibilities for low-risk experimentation), and observability (visibility of student artefacts and learning processes). Combining Koichu et al.'s ecological perspective with Rogers' account of perceived attributes frames

implementation as negotiated, context-dependent work rather than straightforward adoption.

Rather than launching a new enactment, we conduct a retrospective analysis of data from the original design-based cycle with 17-year-old students in Uruguay (Tejera, 2021). Guided by the learning-oriented part of our ToC, this analysis focuses on two strands: (1) coding students' written productions and discussions for the presence and quality of register elements, including which semiotic registers are used and how treatments and conversions are coordinated; and (2) identifying and categorising instances of variational and covariational language, tracing how students describe change and relate changes in one quantity to another. Each task is tagged with a variational task type, allowing us to examine how different task families elicit particular patterns of representation and language. This retrospective work refines the design principles by clarifying which features of task design and orchestration appear critical for supporting covariational reasoning.

These refined design principles then serve as a basis for formulating humble implementation principles that explicitly bridge learning and implementation concerns. Empirically grounded insights about which features must be preserved in enactment are linked to Rogers' perceived attributes and to Koichu et al.'s subprocesses of implementation as ecological disruption. In this way, the design-to-implementation pathway is anchored in both mathematics education theories and implementation-oriented accounts of how innovations travel and transform across contexts.

What is implemented?

The innovation is a three-task GeoGebra-mediated sequence centred on the “filling containers” problem (University of Nottingham, 1990), extended to elicit students' variational arguments. The sequence is organised and delivered via GeoGebra Classroom, enabling teachers to sequence tasks, monitor work in real time, pause for whole-class discussion, and retain artefacts for later analysis. Within this sequence, each task is conceived as part of a variational task family (Caballero-Pérez & Cantoral, 2013), foregrounding particular relationships between changing quantities in complementary ways. The three tasks thus coordinate different descriptions of the filling process, supporting the progressive refinement of students' variational language and arguments.

What is the potential importance and significance?

Deriving implementation principles for this innovation addresses the procedural focus problem by converting a promising design into implementation-ready knowledge and by explaining how future pilots can

be conceptually and empirically evaluated. Recent research on technology-mediated mathematical modelling in calculus has shown how GeoGebra can support students in coordinating real, mathematical, and computational perspectives on optimisation problems (Martínez-Miraval et al., 2025). The work relocates Bach et al.'s (2025) design-to-implementation pathway to an upper-secondary school context focused on variational thinking with a ToC anchored in semiotic registers and variational task families rather than in a tool-use theory.

References

- Bach, C. C., Gregersen, R. M., Højsted, I. H., Pedersen, M. K., & Jankvist, U. T. (2025). Design implementation: The transition from design principles to implementation principles. *Implementation and Replication Studies in Mathematics Education*, 5(1), 48–70. <https://doi.org/10.1163/26670127-bja10025>
- Caballero-Pérez, M., & Cantoral, R. (2013). Una caracterización de los elementos del pensamiento y lenguaje variacional. In R. Flores (Ed.), *Acta Latinoamericana de Matemática Educativa* (Vol. 26, pp. 1197–1205). CLAME.
- Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, 33(5), 352–378. <https://doi.org/10.2307/4149958>
- Duval, R. (2017). *Understanding the mathematical way of thinking – The registers of semiotic representations*. Springer. <https://doi.org/10.1007/978-3-319-56910-9>
- Koichu, B., Sánchez Aguilar, M., & Misfeldt, M. (2021). Implementation-related research in mathematics education: The search for identity. *ZDM – Mathematics Education*, 53(5), 975–989. <https://doi.org/10.1007/s11858-021-01302-w>
- Maher, N., Muir, T., & Chick, H. (2022). Analysing senior secondary mathematics teaching using the Knowledge Quartet. *Educational Studies in Mathematics*, 110(2), 233–249. <https://doi.org/10.1007/s10649-021-10125-1>
- Martínez-Miraval, M., García-Cuéllar, D., Tejera, M., & Curo Cubas, A. (2025). Modelización matemática en temas de cálculo: Dos aproximaciones tecnológicas a un problema de optimización. *Uniciencia*, 39(1), 1–25. <https://doi.org/10.15359/ru.39-1.18>
- Rogers, E. M. (2003). *Diffusion of innovations* (5th ed.). Free Press.
- Tejera, M. (2021). Modelos matemáticos mediados por GeoGebra para el desarrollo del pensamiento variacional. *Reloj de Agua*, 24(1), 39–49.
- University of Nottingham. (1990). *El lenguaje de funciones y gráficas*. Ministerio de Educación y Ciencia. Centro de Publicaciones.