

Implementing a genetic decomposition in a new institutional context: A conceptual replication

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What problem is addressed?

In undergraduate Linear Algebra courses, there is a persistent tendency to reduce the study of linear systems to the mechanical application of the Gaussian elimination algorithm. This procedural focus limits students' ability to interpret structural features of linear systems, such as whether they have a unique solution, infinitely many solutions, or no solution, and to relate these features to the contexts in which systems of equations are used. Dorier's (2000) historical analysis shows that this procedural conception has deep epistemological roots in Euler's work, particularly in what Dorier calls *inclusive dependence*, whereby solvability was associated with determining whether one equation was "included" in another. Although operationally effective, this view limited the understanding of linear systems to algorithmic manipulation for centuries.

The persistence of this algorithmic orientation in contemporary instruction is therefore not incidental; it reflects a historical continuity of a conception that privileges procedural execution over conceptual understanding. For this reason, research in the didactics of Linear Algebra has sought to explain how understanding of linear systems develops. For example, Possani et al. (2010) proposed a genetic decomposition (GD) for

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linear systems, a cognitive model that characterizes the structures and mental constructions needed to conceive systems through the notions of solution set and intersection of solutions. However, there is limited empirical evidence regarding how this GD behaves when transported to different institutional contexts.

Recent literature on conceptual replication (Aguilar, 2020) emphasizes the need to investigate how a theoretical model operates under contextual variation, particularly during its implementation. That is, how the mental constructions anticipated by a GD are promoted when enacted in a regular university course under real instructional constraints. Accordingly, this study asks:

How do the mental constructions anticipated by the GD of Possani et al. (2010) develop during its implementation in a regular Linear Algebra course in a different institutional context, and how do the real conditions of the classroom mediate their emergence?

How is it addressed (theory/methods)?

The study is grounded in three theoretical components. First, it adopts APOS Theory (Arnon et al., 2014), which models mathematical learning as the construction and reorganization of mental structures—actions, processes, objects, and schemas—through mechanisms such as interiorization, coordination, and encapsulation. This perspective makes it possible to characterize the constructions that may be required for understanding linear systems. Within APOS, the genetic decomposition (GD) proposed by Possani et al. (2010) provides the central theoretical reference for describing the constructions involved in conceiving system equivalence as a relation between solution sets.

Second, the study is situated within the literature on conceptual replication in mathematics education (Aguilar, 2020). Replication is understood not as the literal duplication of procedures, but as an investigative process aimed at examining how a theoretical or empirical result behaves when transferred to a context different from the original.

Finally, implementation refers to the systematic study of how a theoretically grounded instructional design operates when enacted under real classroom conditions. Century and Cassata (2016) emphasize that implementation does not consist in verifying whether a design “works,” but in describing the interactions between the design and its contextual conditions: time constraints, available resources, patterns of student collaboration, instructional practices, and institutional culture.

Following the conceptualization proposed by Aguilar (2020) and the implementation framework of Century and Cassata (2016), this study is designed as a conceptual, external, and exploratory replication of the genetic

decomposition for linear systems developed by Possani et al. (2010). The aim is not to revalidate the original GD, but to examine how its theoretical constructions operate when a teaching sequence derived from it is implemented in a different instructional context. In alignment with the research cycle of APOS, the study is situated in the implementation phase, where a previously elaborated GD serves as the basis for designing and enacting instruction.

Data sources consist of (a) written productions generated during the six activities, (b) field notes documenting student-student and student-teacher interactions, and (c) screenshots produced during students' use of GeoGebra. These data support analysis at two levels that are characteristic of a conceptual replication study. First, at a cognitive-constructive level, the analysis examines how the mental constructions anticipated by the original GD (in the APOS sense) emerge, reorganize, or fail to appear when the design is enacted in a different institutional context. Second, at an implementation level, the analysis focuses on how contextual variables—time constraints, available resources, group dynamics, and instructional decisions—mediate the enactment of a GD-based design.

What is implemented/replicated?

The replication involves a scaling-out movement, that is, extending the intervention to a different educational context. In this case, the study was conducted in a regular Linear Algebra course at a Mexican public university with 33 Civil Engineering students, whose academic background differs from that of the participants in Possani et al. (2010). These differences in institutional setting, student profile, and course structure constitute a deliberate contextual variation that allows us to examine the transferability and contextual sensitivity of a GD-based design.

For the implementation, six activities were designed whose structure and sequencing derived explicitly from the original GD. These activities were incorporated into the ordinary dynamics of the course, without creating an artificial or experimental environment, with the purpose of observing how the constructions predicted by the GD emerge or fail to appear under real classroom conditions. Students worked in small groups, engaging in tasks that combined algebraic analysis, work with non-mathematical contexts, exploration with GeoGebra, and the informal production of notation. The instructor maintained the typical role of a regular course: offering support when requested, and providing specific guidance. Each session concluded with collective discussions in which strategies were contrasted, different solution paths were examined, and formal definitions were introduced.

For example, Activity 6 ([geogebra.org/m/pwhurbee#material/fsqgshjt](https://www.geogebra.org/m/pwhurbee#material/fsqgshjt)) was designed to promote coordination among the schemas of equality, equation, and system described in the genetic decomposition of Possani et al. (2010). To this end, the activity employed a GeoGebra applet containing five systems of three linear equations in three variables: two with a unique solution, one with no solution (an inconsistent system), and two with infinitely many solutions, represented geometrically by a line and a plane. The applet allowed students to apply elementary row operations and observe in real time their effect on the graphical representation of the systems. The didactical intention was for students to analyze and discuss what information is preserved under row operations, how this preservation is reflected geometrically in the solution sets, and how the augmented matrix serves as a transformable representation for studying equivalence between systems.

What is the potential importance and significance?

The potential significance of the study lies in examining the transferability and contextual sensitivity of a theoretical model across different instructional settings. Rather than seeking to revalidate the GD of Possani et al. (2010), the study provides evidence about how the mental constructions anticipated by the model emerge, reorganize, or fail to appear when a GD-based instructional design is enacted in a regular university course.

In this sense, the study may contribute not only to the empirical examination of the GD proposed by Possani et al. (2010), but also to a broader understanding of how APOS-based instructional designs behave when transferred from the contexts in which they were originally developed to ordinary classroom settings. This is relevant for distinguishing which aspects of a theoretically grounded design appear to be stable across contexts and which require adaptation when implemented under different institutional and instructional conditions.

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